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A Network-based Scheme for Allocating the Cost of Empty Railcar Movements in Railway Systems

Yao Cheng[1] and Wei-Hua Lin[2]

Abstract—In the freight industry, allocating the cost of empty miles of rail cars to partners in coalition is an important pricing problem. This paper proposes an empty car movement cost allocation scheme based on the game theory that explicitly considers the level of participation and contribution from each partner, the costs generated before and after cooperation, and the overall benefits obtained by each partner due to cooperation. Characterization of cost at each level of cooperation is considered based on the empty miles generated or reduced through cooperation. Compared with other existing models, the proposed cost allocation scheme consistently yields reasonable results, even under the situation in which other models behave poorly. We also show that the proposed pricing scheme can enhance the stability of a coalition such that the subgroup of a coalition will not be better off if the members of the subgroup choose to break away from the coalition.

Index Terms — **Rail Transportation Systems, Game Theory, Pricing, Transportation network, Freight Transportation.**

I. BACKGROUND

THE railroad industry was once a vital link in transportation closely tied with the development of the US economy. With an increased level of congestion on the highway system in the nation and the rapid development in automation and computerization for transportation systems, the role that the railroad industry plays in the national transportation system is growing strong again, especially for the service sector of the US economy. Research in the railroad industry, however, does not seem to keep up with the rapid development in technologies emerging in the development of Intelligence Transportation Systems. In addition, the rail industry today is now facing new challenges and new problems that have not been faced by the industry before.

Over twenty years ago, the practice in multilevel railcar management in the North America railroad network was largely to load railcars at various origins, ship them to their destinations, unload them, and then return each railcar to the loading point from which it was originated. A more efficient practice was initiated in the 1980s among several major rail companies to ship their products under a pooling agreement in which upon unloading the empty cars of each type are treated as part of a common pool, and are redirected to economically convenient loading locations for next loading, rather than being shipped back to the same loading point of origin. This practice requires a much higher level of coordination among participating railroad companies, which has been made possible in the recent years with the rapid development in communication and information technology. There is a trend in the railroad industry moving towards maintaining a universal railcar pool for all participating companies. Nevertheless, even with the increased level of cooperation, a large percentage of the railcar miles are incurred due to the empty car movements. As the level of cooperation further increases in the railroad industry, the fair allocation of the cost of empty railcar movements to each individual company has become an important issue that should be dealt with in conjunction with the role each company plays in cooperation and the cost and benefit incurred to each individual company due to cooperation.

In the past, much research effort in empty rail car movements has been devoted to the operation issues such as the minimization of the total empty car miles. Many algorithms and procedures have been developed to improve the operational efficiency (Misra, 1972, Dejax and Crainic, 1987, Powell, 1987, Haghani 1989, Holmberg *et al*, 1998, Cordeau, *et al*, 1998). Research on allocating fairly the cost of empty miles to each participating partner is quite limited. Traditionally, the return model and the supply model are commonly used in practice. The former is to add empty miles incurred to the loaded trip that generates the empty miles, whereas the later is to add empty miles incurred by a loaded trip to the subsequent trip. In other words, the return model treats an empty car as a commodity that has to be returned by

1 Yao Cheng is graduate student with Department of Systems and Industrial Engineering, The University of Arizona, Tucson, AZ 85721-0020, U.S.A. (email: ycheng@email.arizona.edu).
2. Wei-Hua Lin (corresponding author) is assistant professor with Department of Systems and Industrial Engineering, The University of Arizona, Tucson, AZ 85721-0020, U.S.A. (email: weilin@sie.arizona.edu).

the user and, thus, the current user should cover the total cost for returning. The supply model treats an empty car as a commodity to be supplied to the next shipper and the next shipper is fully responsible for the cost of empty miles incurred in transfer. Another model that has been used is a mixture of the two, which assigns the cost of empty miles to both the current shipper and the subsequent shipper based on the loaded miles or the profits generated by each shipper. Though these cost distribution schemes can work properly under certain operation scenarios, they do not consider in detail the gain and loss incurred in cooperation and subsequently do not ensure the fairness in all situations. Under some circumstances, these models would yield a distribution of costs such that some participants in cooperation would become worse off because of cooperation. Consequently, these participants may decide to withdraw from the pooling agreement system, which could lead to a potential loss to the entire system because of the reduced level of cooperation. In addition to this deficiency, some of the existing models are difficult for implementation on a large scale network. This is especially true for the supply and return models. For a network with many players, it is not easy to distinguish if a portion of empty car miles should be treated as part of a supply trip or as part of a return trip.

This paper proposes a cost-allocation scheme for empty car movements that can be implemented on a large-scale network with many participants. In specific, the proposed cost-allocation scheme will reflect the level of participation in cooperation, the costs generated before and after cooperation, and the benefits obtained by each participant due to cooperation. The paper is organized as follows. In the next section, we will discuss the mathematical formulation of the proposed cost allocation scheme. Sec. 3 discusses the performance of the proposed method by comparing it with those obtained by other models with two simple examples. Sec. 4 presents an algorithm for allocating the empty mile cost to participating companies on a network with n-participating companies and a single depot, where all rail cars must be originated from and returned to after serving all the loaded trips. This is followed by a summary of the research result and a discussion of the future research given in Sec. 5.

II. Mathematical Formulation of the Proposed Scheme to Allocate Costs of Empty Miles to Participating Rail Companies

By treating the corporation among rail companies in sharing railcars as a standard cooperative game, we can reformulate the problem into one that can be solved by the Shapley value method in cooperative game (Shapley, 1953, 1967, 1971). A standard cooperative game is defined as: $\Gamma=\{N,v\}$, in which $N=\{1,2,\cdots,n\}$ is a non-empty finite set of players, representing rail companies and v, with R^{2^n-1} points, is a characteristic function defined on all subsets S (coalitions) of N (grand coalition). The characteristic function must have the following two properties:

1) $v(\varnothing)\equiv 0$, where $\varnothing$ denotes the empty set.
2) $v(S)$ is super-additive, i.e. $v(S\cup T)\geq v(S)+v(T)$, $\forall S,T\subseteq N, S\cap T=\varnothing$.

Assume that the players form the grand coalition by entering one by one in some order and that all $n!$ orderings of the players N are equally likely. Then, the expected marginal contribution made by partner i as it joins coalition can be characterized by:

$$x_i\equiv\varphi_i(N,v)=\sum_{S\in S_i}\frac{(|S|-1)!(n-|S|)!}{n!}[v(S)-v(S-\{i\})] \quad , \quad \forall i=1,\dots,n \tag{1}$$

where S_i denotes all the subsets in N that contain i. $|S|$ denotes the cardinality of the coalition $S\subseteq N$. The collection $\{\varphi_i(N,v)\}_{i\in N}$ is called a set of Shapley value. In Eqn. (1), $v(S)-v(S-\{i\})$ reflects the individual contribution made by i to coalition of S . $v(S)=v(S-\{i\})$ implies that there is no contribution from i.

The payment vector $\boldsymbol{x}=(x_1,\cdots,x_n)\in R^n$ defines for each player $i\in N$ a share x_i which this player gains from the payoff of the coalition. A payment vector $\boldsymbol{x}$ is called a distribution for a cooperative game based on a grand coalition N and has a characteristic function v if $x_i\geq v(\{i\})$, $\forall i=1,2,\dots,n$ and $\sum_{i\in N}x_i=v(N)$. The set of all possible distributions is denoted by $E(v)$. Relation $x_i\geq v(\{i\})$ is the individual rational condition, since it indicates that the payoff gained by any player i is not less than the payoff available to i when i is not part of coalition. Likewise, relation $\sum_{i\in N}x_i=v(N)$ is the collective rational condition. Given distributions $\boldsymbol{x}$ and $\boldsymbol{y}$, and a coalition $S\subseteq N$, $\boldsymbol{x}$ is said to be superior to $\boldsymbol{y}$ on S, denoted by $\boldsymbol{x}\succ_S \boldsymbol{y}$, if $x_i\geq y_i$, $\forall i\in S$ (with at least one strict inequality) and $\sum_{i\in S}x_i\leq v(S)$. A distribution $\boldsymbol{x}$ satisfying this inequality is a feasible distribution for the coalition S. For an $n-$player cooperative game (N,v), the non-dominated distributions in $E(v)$, i.e., the distribution for which there does not exist any other superior distribution with respect to any coalition $S \subseteq N$, is called the core of the game. The core, which is composed of all payment vectors satisfying $\sum_{i\in S} x_i \geq v(S)$, $\forall S \subseteq N$ and $\sum_{i\in N} x_i = v(N)$ (Shapley 1967, 1971), would effectively ensure a fair distribution of costs among any subgroups in the coalition, which would in turn enforce the stability of the coalition.

The merits of the above formulation can be shown by comparing the result obtained by (1) with the results obtained by other models. This will be discussed in the following section with two examples.

III. The Performance of the Proposed Scheme

The results from our preliminary study show that the Shapley value method is very promising in allocating properly the costs of empty cars to all the companies in coalition. It will yield results consistent with those generated by other methods when those methods are appropriate. Most remarkably, it will also generate reasonable results in the situation in which the cost distribution generated by other methods is unreasonable.

A. Comparison with supply and return models

We fist consider an example adopted from a previous study by Davis, 1984. The network for the problem is given in Fig. 1. Nodes A, B, C, and D represent terminals for loading or unloading. Company 1 loads cars at B and moves to C, whereas Company 2 loads cars at D and moves to A. Without cooperation, rail cars will be returned to its previous loading point for each loaded trip, resulting in a total of 1000 empty miles. With cooperation, the rail cars will be moved in a cycle. The total empty miles generated are 600 miles. With a supply model, Company 1 would be allocated a charge equivalent to 400-mile empty movement and Company 2 would be allocated a charge equivalent to 200-mile empty movement. With a return model, Company 1 would be responsible for the 200-mile empty car movement and Company 2 would be responsible for the 400-mile empty car movement. Neither result is reasonable since in this case the benefits obtained by the two companies are exactly the same. The costs to these two companies without cooperation are also the same. Therefore, each company should share the cost of empty miles on an equal basis. By formulating the problem into a two-person game and using the Shapley value method, we can consider the gains at the each level of cooperation as negative costs. We thus have $v(\{1\}) = -500$, $v(\{2\}) = -500$, and $v(\{1,2\}) = -600$. The costs of moving empty cars allocated to Companies 1 and 2 can be determined as follows:

$$\varphi_1(v) = \frac{v(\{1\}) + v(\{1,2\}) - v(\{2\})}{2} = -300\text{, and}$$

$$\varphi_2(v) = \frac{v(\{2\}) + v(\{1,2\}) - v(\{1\})}{2} = -300,$$

indicating that the costs allocated to Companies 1 and 2 are identical.

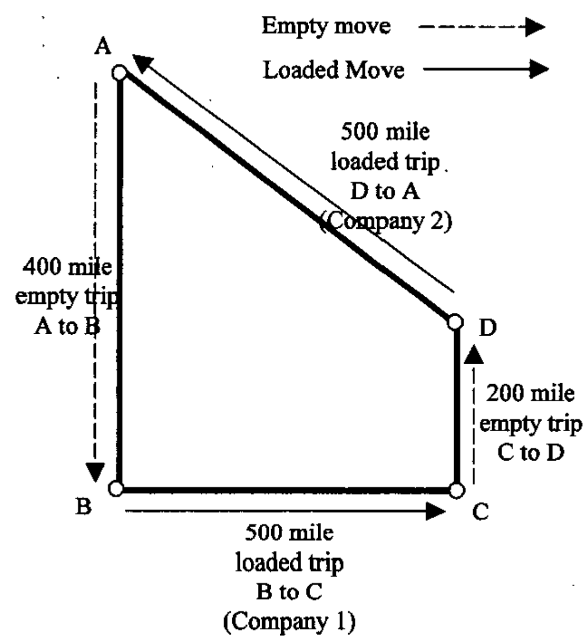

Fig 1: Illustrative example 1 (adopted form Davis 1984): Comparing the result from the proposed scheme with those from return and supply models.

B. Comparison with models based on loaded trips

The example shown here is a comparison between the proposed method and the method that assigns the empty miles proportionally to the loaded trip. The configuration of the network is given in Fig. 2. The length of each segment is defined as follows:

AB = BC = CD = DC = 700; CA = 100, DA = 600.

Company 1 loads cars at A and moves to B. It then loads cars again at B and moves to C. Without cooperation, Company 1 would generate 100 miles of empty car movement from C to A. The whole cycle will start all over again. Company 2 would load at C and then moves to D and unloads at D. it will then return cars to C using path DC, generating a total of 700 empty mile. Thus, the combined empty miles incurred by both companies are 800 miles.

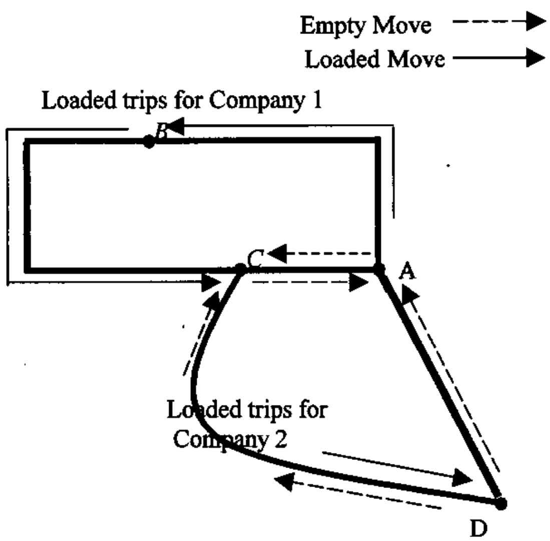

Fig 2: Illustrative example 2: Comparing the result from the proposed scheme with the result from models based on loaded trips.

With cooperation, Companies 1 and 2 would share rail cars. Once Company 1 unloads at C, the car will become available to Company 2. After Company 2 completes its trip from C to D, it will return cars to A instead of C. The combined empty miles are reduced from 800 miles to 600 miles. If we allocate the costs of empty miles based on the length of the loaded trips (1400 and 700), then the costs distributed to Companies 1 and 2 are 400 and 200, respectively. As a result, the cost incurred to Company 1 increases from 100 to 400, indicating that Company 1 becomes worse off due to cooperation. Using the Shapley value method in this case, the costs associated with Companies 1 and 2 before cooperation are $v(\{1\}) = -100$, $v(\{2\}) = -700$, respectively, and the total cost incurred after cooperation is $v(\{1,2\}) = -600$. The costs allocated to Companies 1 and 2 are:

$$\varphi_1(v) = \frac{v(\{1\}) + v(\{1,2\}) - v(\{2\})}{2} = 0 \text{ and}$$

$$\varphi_2(v) = \frac{v(\{2\}) + v(\{1,2\}) - v(\{1\})}{2} = -600,$$

respectively, indicating that Company 2 should bear all the empty mile cost incurred with cooperation. Even so this is still an acceptable distribution to Company 2 because Company 2 is better off with cooperation. Its cost is reduced by 14% because of cooperation. Similarly, Company 1 benefits more significantly from cooperation compared with its base cost. The win-win situation resulting from a fair distribution of costs would effectively keep both companies in the coalition.

IV. Network-Based Formulation and Solution Algorithm and Numerical Examples

In the simple examples shown in the previous section, a reasonable way of cost allocation can also be easily determined by inspection. For a network with many players, solutions are often not very obvious. One of the major complications in applying the Shapley value method is the requirement for characterizing explicitly the costs incurred at all levels of cooperation, which, in turn requires for the knowledge of the costs of cooperation beyond its current form. In reality, however, this type of information is often extremely difficult to obtain. The only information available might be the one associated with the existing form of cooperation. However, one can estimate the potential costs for different form of cooperation based on the existing form of cooperation. A systematic way of decomposing the level of cooperation into subgroups of coalition is needed in order to characterize the cost of cooperation at various levels. This is possible for certain form of cooperation. We show below a solution algorithm that can handle n-players (rail companies in a pooling agreement) in a network with a single depot. For this network, we assume that the railcar requirements for all companies are identical. We also assume that railcars must be originated at the depot and returned to the depot after serving the loaded trips. In addition, we assume that each company serves only a single loaded trip in the network. The algorithm can be described step by step as follows:

[Step 1] Compute c_i, the cost of empty miles generated by Company i when operating independently. c_i is linearly proportional to the empty miles generated in each trip. Construct a distance matrix $\{d_{ij}\}$. d_{ij} is defined as the shortest distance between the destination of a loaded trip served by Company i and the origin of a loaded trip served by Company j. (Note that the matrix can be asymmetric.) Set $m = 1$.

[Step 2] For all subsets $S \subseteq N$ and $|S| = m$, solve the TSP based on $\{d_{ij}\}$ for a tour that departs from the depot, covers all m loaded trips served by S, and then returns to the depot. Calculate $v(S)$ for $\forall S \subseteq N$ and $|S| = m$. $v(\{i\}) = 0$, $\forall i \in N$.

[Step 3] Set $m = m + 1$. If $m \leq |N|$, go to Step 2. Otherwise, calculate $x_i (\forall i \in N)$ defined in Eqn. (1), the benefit that Company i brings to the coalition by its participation.

[Step 4] Compute the cost allocated to Company i, $c_i - x_i$.

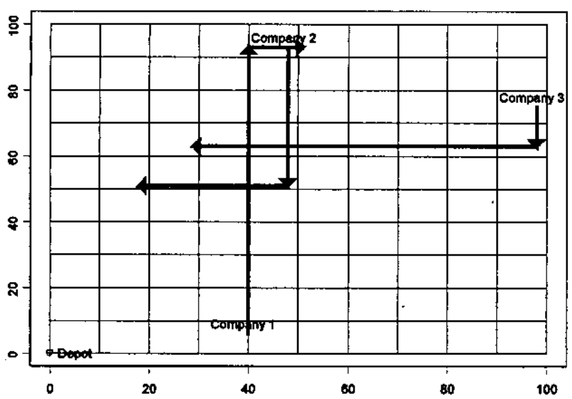

Fig 3: The network with a single depot located at the origin and three loaded trips served by Companies 1, 2, and 3 (The solid line indicts a loaded trip for company i, originated from the location labeled company i)

Cost allocation can also be made simply based on the proportion of the empty miles generated by each company when it operates independently. It can also be based on the length of the loaded trips. The rationale behind that is the companies that demand more services in the coalition should be subject to a larger share of cost when it comes to allocating the cost of empty miles. This, however, may not be desirable because there is no guarantee that the cost would be allocated in a fair way. Moreover, in some cases some companies will be better off by forming a coalition with only a subset of players in the coalition instead even though a grand coalition would reduce the total cost further. Consequently, the grand coalition under such cost allocation schemes tends to be unstable. Cost allocation based on the Shapley value method does not have this undesirable property. This is illustrated in the following network example. Fig. 3 shows a network for three companies, $N = \{1, 2, 3\}$. All trips within the network take place along the coordinate axes. For simplicity, the depot is located at origin (0, 0).

Without cooperation, the empty mile costs to companies 1, 2, and 3 are 190, 210, and 265 units respectively, with a combined cost of 665 units. The total cost under a grand coalition is reduced to 229. Based on the Shapley value method, the costs allocated to companies 1, 2, and 3 are 17, 71, and 141, respectively. The costs allocated to Companies 1, 2, and 3 based on the empty miles generated by each company are 65, 73, and 91, respectively. The two results are similar in terms of ranking. However, Companies 1 and 2 could be better off by forming a coalition themselves without the participation of Company 3. With this, Companies 1 and 2 would pay 56 and 61, respectively, slightly less than the cost they would have been subject to under a three-way coalition. This would make it unattractive for Companies 1 and 2 to remain in the grand coalition. With their departure from the coalition, the cost to Company 3, being operating by itself, will soar to 265 and the overall cost will be increased as well.

The algorithm described above is by no means a realistic representation of the process of operations in freight transportation in the rail system. Nevertheless, the framework considered here, with some modifications and addition of details, could be adapted to other form of operations with different cost functions. For example, instead of considering a single depot, multiple depots might be considered. In this case, algorithms for vehicle routing need to be modified to improve operation efficiency.

V. Conclusions and Future Research

This paper formulates the problem of allocating the costs of empty railcars to the participating rail companies into a network program in which the Shapley value method in game theory for distributing the cost can be adopted to provide a reasonable cost allocation scheme. The proposed method consistently yields reasonable results even under the situation in which other models have performed poorly. The cost function used in this paper is assumed to be a linear function of the empty miles. Studies are ongoing for the use of empirical cost functions that can capture the existing operation strategies and determine the key variables involved in characterizing the costs for various operation settings. Some of the alternative ways of characterizing costs by using different allocation indices will also be considered (Voorde, 1986). We have thus far considered only a single depot. We will extend the result to a network with multiple depots. Currently, each loaded trip is filled with goods from a single company. We will remove this restriction by considering shared loads for a loaded trip and capacity constraints of railcars.

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