**Proceedings of the 8th International
IEEE Conference on Intelligent Transportation Systems
Vienna, Austria, September 13-16, 2005**

A Mathematical Programming Module for Merge Control in System Optimal Dynamic Traffic Assignment

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Abstract— **This paper attempts to enhance the existing linear programming (LP) formulation of the system optimal dynamic traffic assignment SO-DTA problem by treating the priority ratio of a merge junction with a linear programming module equivalent to the non-linear merge component in the cell transmission model. The proposed formulation preserves the proper merge priority rule and can be readily embedded into an LP formulation of the SO-DTA problem for a single destination. It is capable of distinguishing control governed by the roadway geometry and control imposed by the system provider. The other contribution of the paper is that the model proposed also provides an alternative graphical interpretation of the merge rule described in the cell transmission model.**

I. BACKGROUND

THE objectives of dynamic traffic assignment is to predict traffic flow patterns over time and space on a transportation network for a given set of time-varying origin-destination OD demands based on some predefined conditions. The system optimal state is a situation in which vehicles are distributed in a network that minimizes the total vehicle delay. It can be employed to develop effective control strategies for system operations and planning.

The analytical system optimal dynamic traffic assignment SO-DTA models are conventionally formulated as a mathematical programming problem. The pioneering work in this area was due to Merchant and Nemhauser [1] and [2] who formulated the problem as a discrete-time, nonlinear, nonconvex mathematical program for the single destination program. Much of the later work along this line sought to simplify the formulation [3, 4], to develop more efficient solution algorithms [5], and to extend the formulation to handle many-to-many networks [6]. Though models proposed in recent years vary in detail, the modeling approach adopted was fundamentally the same. The underlying traffic flow model uses exit functions or link performance functions in which the exit flow of a link depends only on the content of the link. The exit function could yield flow patterns that would never arise in real life [7]. It was also shown in [8] that the results from models with exit functions differ substantially from those obtained by the kinematic wave model [9, 10] or the LWR model. The LWR model is capable of capturing the dynamic process of physical queues. For the past decade, various models have been proposed [11, 12] to formulate the SO-DTA problem as a linear programming problem based on the cell transmission model [13, 14], which is a discrete version of the LWR model.

Though the analytical SO-DTA models based on the cell transmission model are more realistic in representing traffic dynamics than their counterparts developed with link performance functions, they tend to simplify the treatment of the merge junction in the cell transmission model by assuming system control over vehicles in the entire network, since the merge component in the cell transmission model is inherently nonlinear. This simplification, however, could give rise to merge priorities that are physically difficult or even impossible to realize due to the given roadway configuration.

Merge priorities are sometimes determined by the type of traffic control imposed at merge junctions, such as ramp metering, yield and stop signs, lane barriers, variable message signs, etc. Some are intended to enhance safety and/or to ensure equity in accessing the facility, as opposed to minimizing the total vehicle delay. Other forms of "control" that would affect the merge priority are governed the roadway configuration, such as the merge of vehicles on a freeway in the on-ramp area. When queues are present both on the ramp and on the freeway, it was found that a reasonable merge priority ratio for freeway and ramp traffic would be $(2m-1):1$ for a freeway with m lanes and a ramp with a single lane [15]. Merge priorities resulting from the roadway geometry can be easily captured in a simulation model. It is, however, a challenge to be included in an analytical model with the mathematical programming approach.

Merge priorities on a merge junction could ultimately affect the queuing development process upstream of the merge junction both temporarily and spatially. This can be illustrated with an example. For the merge junction shown in Fig. 1, suppose the capacity for the main branch is c. The capacities for the two merge branches are c_1 and c_2 the slopes of the dashed lines in the Fig, respectively. $c_1 < c$, $c_2 < c$, and $c_1 + c_2 > c$. The merge priority is determined by a merge ratio, which is 1:4. Suppose that queues on branches 1 and 2 are initially Q_1 and Q_2, respectively. For simplicity, we assume that the incoming flows to the two branches are 0. We need to determine the exit flows from the two merge branches to achieve a system optimum condition. Without constraints to enforce the 1:4 ratio, the system optimal

Manuscript submitted on February 21, 2005. This work was supported in part by the PATH program under Grant 1402/B745-01 .

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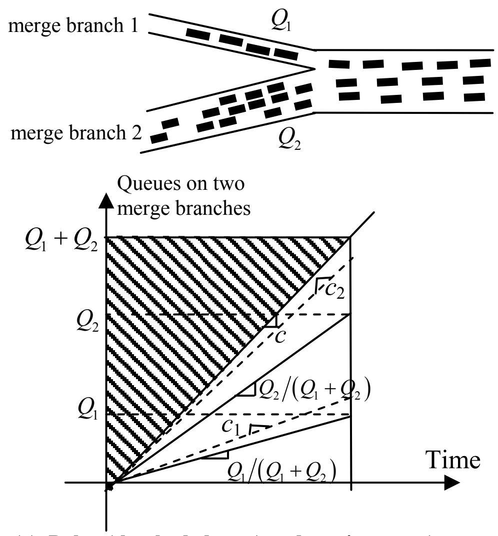
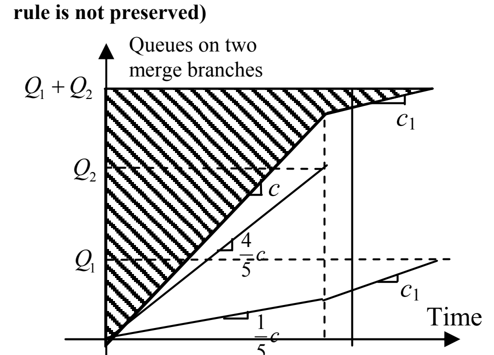
solution would keep the main branch discharging vehicles at the highest rate possible until both queues are dissipated. Therefore, the proportion of flows from the two merge branches should be such that both queues vanish at exactly the same time. The solution can be identified graphically in Fig. 1a with the cumulative vehicle count curve representation. Given *c*, Q_1, and Q_2, the discharge rate for each branch can be determined. Let the flow discharge rate be q_1 and q_2 for merge branches 1 and 2. We then have $q_1 = \frac{Q_1}{Q_1+Q_2}c$ and $q_2 = \frac{Q_2}{Q_1+Q_2}c$. For merge ratio equal to 1:4, branch 1 would operate at $q_1 = \frac{1}{5}c$ before Q_2 clears and $q_1 = c_1$ thereafter. Branch 2 would operate at $q_2 = \frac{4}{5}c$, assuming that $\frac{4}{5}c < c_1$ and $\frac{1}{5}c < c_2$. Q_1 and Q_2 may not clear at the same time, as shown in Fig. 1(b). The total delay in Fig. 1(a) represented by the shaded area is the smallest possible one.



(a) Delay (the shaded area) and service rate (merge priority rule is not preserved)



(b) Delay (the shaded area) and service rate (merge priority rule is preserved)

Fig. 1. The Impact of Merge Control on Total System Delay.

In a SO-DTA model, different levels of control as discussed above should be properly distinguished since they may lead to different queuing patterns. Vehicle holdings should be considered in conjunction with the constraint imposed by the roadway geometry. In this example, we assume for simplicity that both capacities c_1 and c_2 for the two merge branches are inactive. The treatment would be more complicated when we take into consideration the capacity constraints.

In this paper, we present a linear programming formulation for the merge junction capable of distinguishing control at different levels. In particular, our formulation generates a solution that preserves the proper merge priority rule and replicates exactly the feature in the cell transmission model for this particular application. The linear form of our formulation can be integrated into an analytical SO-DTA model for a many-to-one network.

The proposed model, as many other analytical DTA models, is difficult to implement for real-time control. It is, however, useful to serve as a benchmark offline for evaluating the performance of various optimization heuristic developed for traffic control at different levels. Secondly, the proposed model provides an alternative representation of the merge priority rule described in the cell transmission model. In the following section, we will describe in detail our formulation of the merge component. Following that, a numerical example will be presented to demonstrate that the solution of a SO-DTA model can be very different with and without explicitly distinguishing control at different levels at merge junctions.

II. Enhancement to Modeling Merge Junctions

In this section, we show how the merge component in the cell transmission model can be represented by a linear programming module. The network representation in the cell transmission model differs from the traditional link-node representation [14]. In the cell transmission model, freeway segments are represented by a series of cells. The geometry of a cell is characterized by its capacity and density. The length of a cell is chosen such that it takes a single time step to traverse a cell at the free flow speed. The following is a notation list used for model formulation. The merge cell *i* is shown in Fig. 2, which has two companion cells, indexed by $u_1(i)$ and $u_2(i)$, representing the two upstream merge branches.

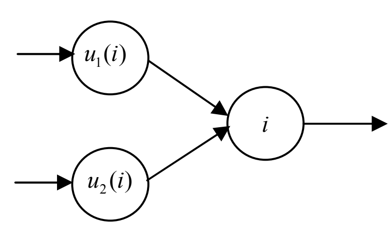


Fig. 2. Description of the merge cell the cell labeled *i*.

<u>Decision variables:</u>

$n_i(t)$: number of vehicles in cell i at time t .

$y_i(t)$: number of vehicles entering cell i in $[t, t+1)$.

$z_i(t)$: number of vehicles leaving cell i in $[t, t+1)$.

$p_i(t)$: dummy variables for merge priority.

$q_i(t)$: dummy variables for merge priority.

Endogenous variables:

α: wave coefficient $\left(=\frac{v_f}{w}\right)$, where v_f is free flow speed and w is wave speed.

$Q_i(t)$: capacity maximum number of vehicles that can enter cell i in $[t, t+1)$.

$N_i(t)$: jam density maximum number of vehicles that can reside in cell i in $[t, t+1)$.

λ_i: the merge priority for vehicles entering cell i from cells $u_1(i)$ and $u_2(i)$.

P_1, P_2: Penalty terms.

In many analytical models based on the cell transmission model, flows exiting from the two merge branches, $z_{u_1(i)}(t)$, $z_{u_2(i)}(t)$, and entering into the merge cell *i*, $y_i(t)$, are usually modeled by the following set of constraints:

$$z_{u_1(i)}(t) + z_{u_2(i)}(t) \le Q_i(t) \quad \forall t. \qquad (1a)$$

$$z_{u_1(i)}(t) + z_{u_2(i)}(t) \le \alpha\left(N_i(t) - n_i(t)\right) \quad \forall t. \qquad (1b)$$

$$z_{u_1(i)}(t) \le Q_{u_1(i)}(t) \quad \forall t. \qquad (1c)$$

$$z_{u_2(i)}(t) \le Q_{u_2(i)}(t) \quad \forall t. \qquad (1d)$$

$$z_{u_1(i)}(t) \le n_{u_1(i)}(t) \quad \forall t. \qquad (1e)$$

$$z_{u_2(i)}(t) \le n_{u_2(i)}(t) \quad \forall t. \qquad (1f)$$

$$y_i(t) = z_{u_1(i)}(t) + z_{u_2(i)}(t) \quad \forall t. \qquad (1g)$$

The constraints above are a simplification of the network model in the cell transmission model. They do not capture the merge priority that is explicitly described in the model. In the cell transmission model, for a merge junction represented by cell i and two merge cells $u_1(i)$ and $u_2(i)$ shown in Fig. 3, if cell i has sufficient room to receive all vehicles from the two merge branches, then all vehicles residing in these two cells should advance. The actual sending and receiving flows can be calculated by [14]:

$$z_{u_1(i)}(t) = S_{u_1(i)}(t) \text{ ,and } z_{u_2(i)}(t) = S_{u_2(i)}(t),$$
$$\text{if } R_i(t) > S_{u_1(i)}(t) + S_{u_2(i)}(t); \qquad (2a)$$

where $S_i(t)$ and $R_i(t)$ are the maximum sending and receiving flows at time *t*, respectively, calculated for cell i.

If cell i does not have enough room to admit all the vehicles from the two upstream merge cells, then flows advancing from the two upstream cells should follow the rules below as defined in the cell transmission model:

$$z_{u_1(i)}(t) = \text{mid}\left\{S_{u_1(i)}(t), R_i(t) - S_{u_2(i)}(t), \frac{1}{1+\lambda_i}R_i(t)\right\} \qquad (2b)$$

and $z_{u_2(i)}(t) = \text{mid}\left\{S_{u_2(i)}(t), R_i(t) - S_{u_1(i)}(t), \frac{\lambda_i}{1+\lambda_i}R_i(t)\right\} \qquad (2c)$

$$\text{if } R_i(t) \le S_{u_1(i)}(t) + S_{u_2(i)}(t);$$

In both cases, the flow into cell i is $y_i(t) = z_{u_1(i)}(t) + z_{u_2(i)}(t)$. The above equations are non-linear. In the following, we propose an alternative version of the merge constraints equivalent to Equation (2) that can be easily linearized. Consideration reveals that in order to preserve the merge priority, the flow entering cell i from the two merge cells conditioning on the number of vehicles inside the two merge branches should satisfy the following two conditions: (1) The total flow advanced, i.e. $z_{u_1(i)}(t) + z_{u_2(i)}(t)$, is maximized and (2) the absolute value of $z_{u_1(i)}(t) - \lambda_i z_{u_2(i)}(t)$ is minimized.

Condition (1) ensures that a maximum flow, bounded by R_i, is sent from the two merge branches. This would eliminate the unintended vehicle holding. Condition (2) determines the proper flow mix from the two merge branches. Clearly, condition 1 should be enforced on top of condition (2). The resulting solution can be shown with the diagram given in Fig. 3. As shown in the Figure, the merge priority line has a slope of λ_i, which governs the flow mix that enters the merge cell from the two merge branches when $R_i(t) \le S_{u_1(i)}(t) + S_{u_2(i)}(t)$. The area for the maximum sending flow for this condition is to the left side of the receiving flow line as shown in the figure. It can be partitioned into three regions, corresponding to (a) $S_{u_1(i)}(t) \ge \frac{R_i(t)}{1+\lambda_i}$, $S_{u_2(i)}(t) \ge \frac{\lambda_i R_i(t)}{1+\lambda_i}$, (b) $S_{u_1(i)}(t) < \frac{R_i(t)}{1+\lambda_i}$, $S_{u_2(i)}(t) \ge \frac{\lambda_i R_i(t)}{1+\lambda_i}$, and (c) $S_{u_1(i)}(t) \ge \frac{R_i(t)}{1+\lambda_i}$, $S_{u_2(i)}(t) < \frac{\lambda_i R_i(t)}{1+\lambda_i}$, as shown in Fig. 3. The mapping from the maximum sending flow to the actual sending flow based on the cell transmission model is represented by the two dots connected with an arrow line in each region. In region a, the solution is unique. All sending flow mix e.g. S_1 and S_2 as shown in the figure in that region is mapped to the point where the receiving line and the priority line intercepts. In regions b and c, the number of feasible solutions for a sending flow mix are infinite as indicated in the figure. The realizable sending flow mix satisfying all equations defined in the cell transmission model is unique. In all cases, the solution is the one with the smallest $|\Delta| = \left|z_{u_2(i)}(t) - \lambda_i z_{u_1(i)}(t)\right|$ as shown in the figure. When $R_i(t) \ge S_{u_1(i)}(t) + S_{u_2(i)}(t)$, the sending flow and the solution are both in region (d), all vehicles can advance since the capacity is greater than the demand.

For conditions (1) and (2) to be satisfied in all four regions, the objective function of an equivalent mathematical programming formulation can be expressed as:

$$\max \quad P_1\left[z_{u_1(i)}(t) + z_{u_2(i)}(t)\right] - P_2\left|z_{u_2(i)}(t) - \lambda_i z_{u_1(i)}(t)\right|$$

where P_1 and P_2 are penalty terms and $P_1 >> P_2$ to ensure that condition 1 is preserved on top of condition (2). The first term, corresponding to condition (1), ensures that the actual sending flow is equal to the maximum receiving flow, and the second term, corresponding to condition 2, keeps the flow mix closest to the priority line. The linear programming formulation for the merge junction is:

[LP1]

$$\max P_1\left[z_{u_1(i)}(t)+z_{u_2(i)}(t)\right]-P_2\left[p_i(t)+q_i(t)\right]$$

subject to

$$z_{u_1(i)}(t)+z_{u_2(i)}(t)\le Q_i(t) \quad \forall t. \qquad (3a)$$

$$z_{u_1(i)}(t)+z_{u_2(i)}(t)\le \alpha\left(N_i(t)-n_i(t)\right) \quad \forall t. \qquad (3b)$$

$$z_{u_1(i)}(t)\le Q_{u_1(i)}(t) \quad \forall t. \qquad (3c)$$

$$z_{u_2(i)}(t)\le Q_{u_2(i)}(t) \quad \forall t. \qquad (3d)$$

$$z_{u_1(i)}(t)\le n_{u_1(i)}(t) \quad \forall t. \qquad (3e)$$

$$z_{u_2(i)}(t)\le n_{u_2(i)}(t) \quad \forall t. \qquad (3f)$$

$$z_{u_2(i)}(t)-\lambda_i z_{u_1(i)}(t)=p_i(t)-q_i(t) \quad \forall t. \qquad (3g)$$

$$y_i(t)=z_{u_1(i)}(t)+z_{u_2(i)}(t) \quad \forall t. \qquad (3h)$$

The variables in the above constraints are all non-negative.

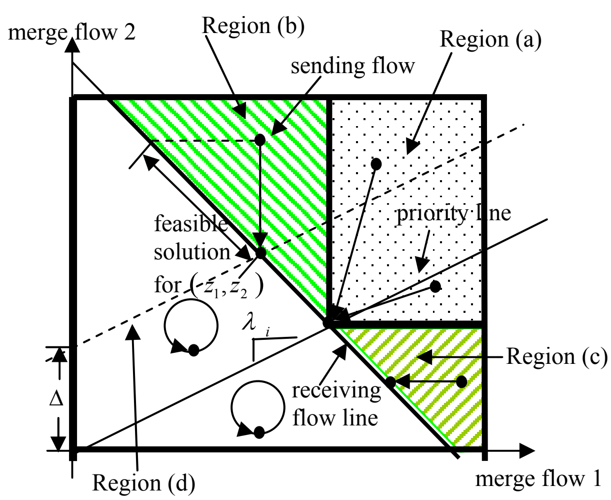


Fig. 3: A diagram for determining the sending flow at merge junctions

We show now that the above formulation is equivalent to the nonlinear model given in Daganzo 1994. We can prove the equivalence between the two by exhausting all possible cases. By making use of the diagram shown in Fig. 3, we only need to consider four cases. We define the sending flow in the same way as that defined in the cell transmission model. The sending flows for merge branches 1 and 2 can be defined as $S_{u_1(i)}(t)=\min\left\{Q_{u_1(i)}(t),n_{u_1(i)}(t)\right\}$ and $S_{u_2(i)}(t)=\min\left\{Q_{u_2(i)}(t),n_{u_2(i)}(t)\right\}$, respectively. The receiving flow, $R_i(t)$, is defined by constraints (3a) and (3b), i.e., $R_i(t)=\min\left\{Q_i(t),\alpha\left(N_i(t)-n_i(t)\right)\right\}$. Together with constraints (3a) and (3b), the actual flow should then satisfy $z_{u_1(i)}(t)\le S_{u_1(i)}(t)$ represented by (3c) and (3e), $z_{u_2(i)}(t)\le S_{u_2(i)}(t)$ represented by (3d) and (3f). The remaining constraints are used for preserving a proper merge priority. The four cases corresponding to regions (a) to (d) shown in the Figure are as follows:

[Case 1] $S_{u_1(i)}(t)\ge \dfrac{R_i(t)}{1+\lambda_i}$ and $S_{u_2(i)}(t)\ge \dfrac{\lambda_i R_i(t)}{1+\lambda_i}$, corresponding to region (a).

The optimal solution to [LP1] is: $z_{u_2(i)}(t)=\dfrac{\lambda_i}{\lambda_i+1}R_i(t)$, $z_{u_1(i)}(t)=\dfrac{1}{\lambda_i+1}R_i(t)$, $p_i(t)=q_i(t)=0$. The objective function is maximized at $z^*=P_1R_i(t)$. This is the case we have $\Delta=0$, since the solution falls exactly on the priority line as shown in Fig. 3. In order to show that the optimal solution is indeed equivalent to the one given by the cell transmission model, we need to show that $\dfrac{\lambda_i}{\lambda_i+1}R_i(t)$ falls between $R_i(t)-S_{u_1(i)}(t)$ and $S_{u_2(i)}(t)$ as in (2c). By definition, we have $\dfrac{\lambda_i}{\lambda_i+1}R_i(t)\le S_{u_2(i)}(t)$. Since $S_{u_1(i)}(t)\ge \dfrac{R_i(t)}{1+\lambda_i}$, we have $R_i(t)-S_{u_1(i)}(t)\le R_i(t)-\dfrac{1}{\lambda_i+1}R_i(t)=\dfrac{\lambda_i}{\lambda_i+1}R_i(t)$. It follows that $z_{u_2(i)}(t)=\dfrac{\lambda_i}{\lambda_i+1}R_i(t)$. Likewise, we can show that $z_{u_1(i)}(t)=\dfrac{1}{\lambda_i+1}R_i(t)$ is the mid term of $R_i(t)-S_{u_2(i)}(t)$, $\dfrac{1}{\lambda_i+1}R_i(t)$, and $S_{u_1(i)}(t)$. Thus, the solution obtained from the linear programming module [LP1] agrees with the solution from the merge component in the cell transmission model.

[Case 2] $S_{u_1(i)}(t)\le \dfrac{R_i(t)}{1+\lambda_i}$ and $S_{u_2(i)}(t)\ge \dfrac{\lambda_i R_i(t)}{1+\lambda_i}$, and $S_{u_1(i)}(t)+S_{u_2(i)}(t)\ge R_i(t)$, corresponding to region (b).

In this case, one of the two constraints, (3a) or (3b), must be binding. Thus, the first term in the objective function is maximized at $P_1R_i(t)=\min\{Q_i(t),\alpha(N_i(t)-n_i(t))\}$. In order to make the second term as small as possible, we need to keep the absolution value of $z_{u_2(i)}(t)-\lambda_i z_{u_1(i)}(t)$ as small as possible. Since in region (b) $z_{u_2(i)}(t)-\lambda_i z_{u_1(i)}(t)\ge 0$, the optimal solution is thus to keep $z_{u_1(i)}(t)$ as large as possible. This can be achieved by making $z_{u_1(i)}(t)$ binding at its upper bound, i.e., $z_{u_1(i)}(t)=S_{u_1(i)}(t)$ and $z_{u_2(i)}(t)=R_i(t)-S_{u_1(i)}(t)$. The objective function is maximized at $z^*=P_1R_i(t)-P_2\left(R_i(t)-S_{u_1(i)}(t)(1+\lambda_i)\right)$. In the extreme case, when $S_{u_1(i)}(t)=\dfrac{R_i(t)}{1+\lambda_i}$ and $S_{u_2(i)}(t)=\dfrac{\lambda_i}{1+\lambda_i}R_i(t)$, we have $z^*=P_1R_i(t)$. The result is the same as that in case 1. In this case, we need to show that $S_{u_1(i)}(t)$ is between $R_i(t)-S_{u_2(i)}(t)$ and $\dfrac{1}{\lambda+1}R_i(t)$. From the diagram in

Fig. 3, $S_{u_1(i)}(t) \le \frac{1}{\lambda+1} R_i(t)$. $S_{u_1(i)}(t) \ge R_i(t) - S_{u_2(i)}(t)$. Thus, $S_{u_1(i)}(t)$ is indeed the mid term. It follows that the exit flow from cell $u_1(i)$ is $z_{u_1(i)}(t) = S_{u_1(i)}(t)$. Similarly, we can show that $z_{u_2(i)}(t) = R_i(t) - S_{u_1(i)}(t)$. The solution obtained is consistent with (2c).

[Case 3] $S_{u_1(i)}(t) \ge \frac{R_i(t)}{1+\lambda_i}$, $S_{u_2(i)}(t) \le \frac{\lambda_i R_i(t)}{1+\lambda_i}$, and $S_{u_1(i)}(t) + S_{u_2(i)}(t) \ge R_i(t)$, corresponding to region (c). This case is symmetric to Case 2.

[Case 4] $S_{u_1(i)}(t) + S_{u_2(i)}(t) \le R_i(t)$, corresponding to region (d). The solution is $z_{u_1(i)}(t) = S_{u_1(i)}(t) = \min\{Q_{u_1(i)}(t), n_{u_1(i)}(t)\}$ and $z_{u_2(i)}(t) = S_{u_2(i)}(t) = \min\{Q_{u_2(i)}(t), n_{u_2(i)}(t)\}$. It is consistent with (2a). Therefore, in all four cases, the solutions from the linear module [LP1] are consistent with the nonlinear merge component defined in the cell transmission model.

III. A NUMERICAL EXAMPLE

We present a numerical example here to show that the solution of SO-DTA models with a mathematical programming formulation can be very different with and without distinguishing control at different levels and by different parties. We use the nine-cell one-to-one network of Fig. 4, with one diverge junction and one merge junction. The capacity and density for each cell are given in the parenthesis in Fig. 4, with the first number denoting capacity and the second number denoting density. The capacity for cell 6 is reduced to 0.2 from time step 5 to time step 7. The demand for the first 8 time slices is 6 units/time unit. The merge priority ratio is 3:1 for cells 4 and 7, merging into cell 8. We also assume that the wave coefficient $\alpha = 1$.

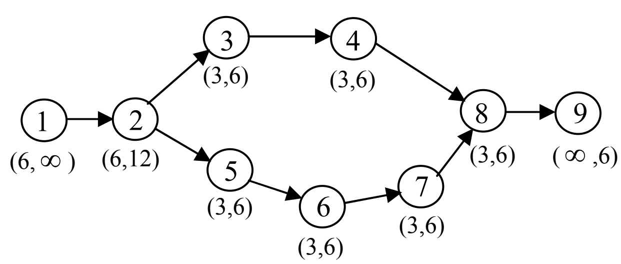


Fig. 4. The example network

We compare the results obtained from two different cases. In case 1, we solve the problem in a conventional way in which merge priority and the unintended vehicle holdings are not explicitly treated. In case 2, we make use of the linear programming module discussed in this paper. The problem was solved by the CPLEX Linear Optimizer. The results are displayed in Tables I and II. Rows 1 to 9 correspond to cells 1 to 9. In each entry of the two tables, two values are displayed as the output of the model. The first value is the number of vehicles in cell i at time t, $n_i(t)$, and the second is the outflow $z_i(t)$, and

TABLE I
SOLUTION 1 (WITHOUT PRESERVING MERGE PRIORITY RULES)

	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9	t=10
1	0.00 0.00	6.00 6.00	6.00 6.00	6.00 6.00	6.00 6.00	6.00 6.00	6.00 3.20	8.80 3.00	5.80 0.00	5.80 5.80
2	0.00 0.00	0.00 0.00	6.00 6.00	6.00 6.00	6.00 6.00	6.00 3.20	8.80 3.00	9.00 3.40	8.60 3.00	5.60 0.00
3	0.00 0.00	0.00 0.00	0.00 0.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 0.00	6.00 0.00	6.00 3.00
4	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 2.40	0.60 0.00	0.60 0.60
5	0.00 0.00	0.00 0.00	0.00 0.00	3.00 3.00	3.00 0.20	5.80 0.20	5.80 0.20	5.60 3.00	3.00 2.40	3.60 0.00
6	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	3.00 0.20	3.00 0.20	3.00 0.20	3.00 3.00	3.00 3.00	2.40 2.40
7	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.20 0.00	0.40 0.00	0.60 0.60	3.00 3.00	3.00 2.40
8	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00
9	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00

	t=11	t=12	t=13	t=14	t=15	t=16	t=17	t=18	t=19	t=20
1	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00
2	11.4 3.00	8.40 0.00	8.40 5.40	3.00 3.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00
3	3.00 3.00	3.00 3.00	0.00 0.00	3.00 3.00	3.00 0.00	3.00 0.00	3.00 3.00	0.00 0.00	0.00 0.00	0.00 0.00
4	3.00 3.00	3.00 0.00	6.00 3.00	3.00 3.00	3.00 0.00	3.00 3.00	0.00 0.00	3.00 3.00	0.00 0.00	0.00 0.00
5	3.60 0.00	3.60 3.00	0.60 0.00	3.00 3.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00
6	0.00 0.00	0.00 0.00	3.00 3.00	0.00 0.00	3.00 0.00	3.00 3.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00
7	3.00 0.00	3.00 3.00	0.00 0.00	3.00 0.00	3.00 3.00	0.00 0.00	3.00 3.00	0.00 0.00	0.00 0.00	0.00 0.00
8	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	0.00 0.00
9	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00

The solution obtained by using the conventional model shown in Table I exhibits vehicle holdings in a number of places. For example, at t = 9, cell 1 has 5.80 units of vehicles and its downstream cell has 8.60 units of vehicles. The exit flow from cell 1 is 0. According to the cell transmission model, the exit flow from cell 1 at $t = 9$ should be 3.40 (i.e., $\min\{5.80, 6.00, 6.00, 12.00 - 8.60\}$) units instead. Thus, 3.40 units of vehicles were held for an additional time step in cell 1. Table 1 also shows that when one does not explicitly specify the 3:1 merge priority ratio, the merge priority sometimes behaves like all-or-nothing assignment. In some situations, only vehicles from one of the merge cells advance when queues are present in both cells.

The solution generated from the proposed formulation (see Table II) exhibits no vehicle holdings. The merge priority at the merge junction is preserved as specified. When queues are present at both of the merge cells, the 3:1 merge ratio is well maintained for the vehicles coming from cells 4 and 7 (see the entries for corresponding cells for $t \ge 9$).

Interestingly, the solutions obtained with and without considering preserving merge priority rules also affect the route choice decisions. For solution 1, 64% of the vehicles choose the route 1-2-3-4-8-9. For solution 2, only 45% of the vehicles choose the same route.

TABLE II

SOLUTION 2 (PRESERVING MERGE PRIORITY RULES)

	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9	t=10
1	0.00	6.00	6.00	6.00	6.00	6.00	6.00	8.80	5.60	2.60
	0.00	6.00	6.00	6.00	6.00	6.00	3.20	3.20	3.00	2.60
2	0.00	0.00	6.00	6.00	6.00	6.00	8.80	8.80	9.00	7.90
	0.00	0.00	6.00	6.00	6.00	3.20	3.20	3.00	4.10	3.00
3	0.00	0.00	0.00	3.00	3.00	3.00	3.00	3.20	3.20	1.50
	0.00	0.00	0.00	3.00	3.00	3.00	2.80	2.80	2.80	0.75
4	0.00	0.00	0.00	0.00	3.00	3.00	3.20	3.20	3.20	5.25
	0.00	0.00	0.00	0.00	3.00	2.80	2.80	2.80	0.75	0.75
5	0.00	0.00	0.00	3.00	3.00	5.80	5.80	5.80	3.00	3.00
	0.00	0.00	0.00	3.00	0.20	0.20	0.20	3.00	3.00	3.00
6	0.00	0.00	0.00	0.00	3.00	3.00	3.00	3.00	3.00	3.00
	0.00	0.00	0.00	0.00	0.20	0.20	0.20	3.00	3.00	2.25
7	0.00	0.00	0.00	0.00	0.00	0.20	0.20	0.20	3.00	3.75
	0.00	0.00	0.00	0.00	0.00	0.20	0.20	0.20	2.25	2.25
8	0.00	0.00	0.00	0.00	0.00	3.00	3.00	3.00	3.00	3.00
	0.00	0.00	0.00	0.00	0.00	3.00	3.00	3.00	3.00	3.00
9	0.00	0.00	0.00	0.00	0.00	0.00	3.00	3.00	3.00	3.00
	0.00	0.00	0.00	0.00	0.00	0.00	3.00	3.00	3.00	3.00

	t=11	t=12	t=13	t=14	t=15	t=16	t=17	t=18	t=19	t=20
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	7.50	4.50	2.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3.00	2.25	2.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	5.25	5.25	4.50	3.75	3.00	2.25	1.50	0.75	0.00	0.00
	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.00	0.00
5	3.00	3.25	3.75	3.75	1.50	0.00	0.00	0.00	0.00	0.00
	2.25	2.25	2.25	2.25	1.50	0.00	0.00	0.00	0.00	0.00
6	3.75	3.75	3.75	3.75	3.75	3.00	0.75	0.00	0.00	0.00
	2.25	2.25	2.25	2.25	2.25	2.25	0.75	0.00	0.00	0.00
7	3.75	3.75	3.75	3.75	3.75	3.75	3.75	2.25	0.00	0.00
	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	0.00	0.00
8	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	0.00
	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	0.00
9	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00

IV. CONCLUSION

This paper presents a modeling component capable of capturing more realistically traffic dynamics in merge junctions based on the cell transmission model. The model departs from many existing analytical models formulated in the past in that it does not assume system control over every part of the network. Our formulation enhances realism in modeling merge junctions by specifically treating the merge priority governed by control other than for the purpose of system optimum as a set of constraints. We show in the paper that the solution obtained from our formulation is consistent with that from the nonlinear version of the merge component in the cell transmission model. The linear form of our formulation makes it possible for the proposed model to be integrated into an analytical SO-DTA model formulated as a linear programming problem. We demonstrated with a numerical example that the solution of the SO-DTA problem can be different with and without considering explicitly the different levels of control in a network. As a byproduct, the paper has also provided an alternative representation of traffic dynamics at merge junctions described in the cell transmission model.

The discussion and formulation of the LP model for the SO-DTA problem in this paper is limited to a many-to-one network. More work will be required to extend the model to address the traffic flow issues on a network with multiple origins and multiple destinations, a multi-commodity problem in which vehicles on the road need to be distinguished by their destinations.

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