Experimental Studies of Sequential Selection and Assignment with Relative Ranks

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Keywords: Selection and Assignment, Secretary Problem, Optional Stopping, Sequential Search

JEL Classification: D83, C44, C61

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Abstract

We study a class of sequential selection and assignment problems, where a decision maker (DM) must sequentially assign applicants to positions in an attempt to minimize total expected cost. In modeling this class of problems, we assume that the DM is only informed of the rank of an applicant relative to the applicants that she previously observed and assigned. We report the results of three experiments designed to study sequential assignment behavior. In comparing the aggregate results from all three experiments to the optimal decision policy that we present and discuss, we identify a systematic bias to assign early applicants to overly costly positions, but with experience subjects approach, though not reach, optimal assignments.
1. Introduction

Derman, Lieberman, and Ross (1972) formulated the following sequential stochastic assignment model. A decision maker (DM) observes a sequence of \( n \) jobs, one at a time, for which there are \( m \) machines available. Associated with each job \( j, j = 1, 2, \ldots, n \), is a value, which is a random variable \( X_j \) that takes on the values \( x_j \). When a job arrives and its value \( x_j \) is revealed, the DM must decide whether or not to assign it to one of the available machines (selection). If she decides to do so, she must next decide which of the \( m \) machines to assign it to (assignment). Assignment decisions are irrevocable; once a job is assigned it may not be reassigned to another machine. Costs or rewards reflect the quality of the match between the type of job (which is revealed upon observation) and the type of machine (which is known \textit{a priori}). The objective criterion is the minimization of total expected cost.

This formulation is quite general; it describes the essential characteristics of sequential selection and assignment problems in other domains. For example, the \( m \) “machines” may correspond to \( m \) interviewers who differ from one another in their experience, personality, gender, etc., and the \( n \) “jobs” to \( n \) applicants who also differ from one another on several attributes and are observed by the DM one at a time. The DM seeks to achieve the best match between the interviewers and applicants. For another example, the DM may correspond to a plant manager, the \( m \) machines to \( m \) subordinates, and the \( n \) jobs to \( n \) projects that arrive one at a time. In assigning projects to subordinates, the manager may wish to maximize the fit between the duration, importance, difficulty, or other requirements of the projects and the qualifications, expertise, and experience of the available subordinates. David and Yechiali (1985) have recognized yet another application of the model to the allocation of kidney transplants. In this application, there are \( m \) candidates for kidney transplants and \( n \) kidneys that become available one at a time. Kidneys and candidates have their own types (e.g., tissue type of the candidate and
of the organ donor). The candidate types are known ahead of time, but the organ types are only revealed upon their arrival. Once a kidney is assigned, the reward is assumed to be the product of the kidney’s and the candidate’s types. The DM’s objective is to maximize the total expected reward (Su & Zenios, 2002). In our experimental set-up, which we describe in detail in Section 3, we used yet another example of sequential triage assignment decisions. The $m$ machines correspond to $m$ hospitals, the $n$ jobs to $n$ injured patients that wait sorting and assignment, and the DM to a triage officer who must sequentially assign patients to hospitals. Each patient $j$ is assigned a score $x_j$ that measures the severity of his injuries, and each hospital $k$, $k=1, 2 \ldots, m$, has an associated cost, $b_k$, that represents the quality of care, distance of the hospital from the DM, or some combination of the above. The DM’s objective is to assign patients to hospitals, one at a time, in an attempt to minimize the total expected cost over the $n$ patients, $\sum_{j=1}^{n} x_j b_k$.

The sequential assignment model of Derman et al. has been extended in different directions. Albright and Derman (1972) studied the asymptotic behavior of the optimal assignment policy derived by Derman et al. Albright (1974) considered an analogue of the discrete time model in continuous time, where jobs are assumed to arrive according to a non-homogenous Poisson process. Sakaguchi (1984) further considered the case in which the number of jobs is unknown, and Kennedy (1986) studied yet another case of dependent (rather than random) job arrivals. For more recent extensions and applications, see e.g., David and Yechiali (1990), David (1995), and Su and Zenios (2002).

In deriving the optimal decision policy for their class of sequential assignment problems, Derman et al. assumed that the $n$ jobs are compared to one another in terms of a single dominant attribute whose distribution function is known to the DM before the assignments commence. In particular, they assumed that associated with each job $j$ is a single $X_j$, which is an i.i.d. variable
randomly drawn from a common univariate probability distribution \( f(x | \theta) \) with parameters \( \theta \). Their approach is known as the full information approach (Gilbert & Mosteller, 1966).

In Section 2 we discuss what we consider to be the limiting features of this approach. To do so, we present and discuss the simpler sequential observation and selection problem—often referred to as sequential search or optional stopping—and characterize the optimal decision policy derived under the full information assumption. Our criticism of the full information approach in this case applies ipso facto to the optimal decision policy of the more complicated sequential assignment problem. Following Chun and Sumichrast (in press, hereafter CS), we describe in Section 3 the no information approach to the sequential assignment problem, in which the assignment decision of the \( j \)th job is solely based on its relative rank among the jobs observed and assigned thus far. (See also Hornberger & Ahn, 1997, who present the no information approach for deriving optimal decision policies for the assignment of transplant organs.) Under the no information approach, the optimal policy for the sequential assignment problem is an extension of the celebrated secretary problem (see, e.g., Freeman, 1983; Ferguson, 1989; Gilbert & Mosteller, 1966; Samuels, 1991) that also assumes a parameter-free information structure. Therefore, Section 3 starts with a brief presentation of the classical secretary problem (CSP), and continues with a formulation of the sequential assignment problem based on relative ranks. In Section 4, we describe and illustrate the optimal decision policy for the no information case, which is due to CS, that applies dynamic programming to numerically compute the optimal assignment of each job \( j \). Sections 5 and 6 present three experiments designed to study selection and assignment decisions of financially motivated subjects in a computer-controlled task. We focus on the comparison of the subjects’ assignment decisions to those dictated by the optimal decision policy. Implications of the experimental results and future research directions are discussed in Section 7.
2. Limiting Features of the Full Information Approach

To illustrate the shortcomings of the full information approach, we apply it to the simpler case of searching for the best of \(n\) objects. A sample of \(n\) observations are drawn from a population with continuous cumulative distribution \(F(\theta)\) with parameters \(\theta\). Knowing \(F(\theta)\) and \(n\), the DM is informed of the value of each observation, whereupon she must choose whether or not to select this observation. Selecting an observation terminates the search. The DM’s objective is to maximize the probability of selecting the largest observation in the sample. Because the distribution \(F(\theta)\) is known exactly and the largest observation in the sample remains the largest under all monotone transformations of its variable, there is no loss of generality in assuming that \(F(\theta)\) is the standard uniform \(F(\theta)=x\) and density function \(f(x|\theta)=1\) on \(0 \leq x \leq 1\) (Gilbert & Mosteller, 1966).

Gilbert and Mosteller showed that the optimal decision policy consists of a vector \(s=(s_1, s_2, \ldots, s_n)\) of cutoff values satisfying \(s_{j+1} \geq s_j\). These cutoff values are computed recursively. The DM should stop the search and take the \(j^{th}\) observation if \(x_j \geq s_j\), and continue, otherwise. As the cutoff values are nonincreasing in the draws, the optimal policy allows the DM to pass over an observation at stage \(j\) but stop and select an observation with a smaller value at a later stage \(j'\), where \(j' > j\). Selected values of the probability of winning, \(P(\text{win})\), are presented in Table 7 of Gilbert and Mosteller. For example, \(P(\text{win})=0.609, 0.594,\) and \(0.587\) for \(n=10, 20,\) and \(40\), respectively, with \(P(\text{win})\) converging from above to \(0.5802\).

The full information approach is rather appealing as it yields point predictions that are experimentally testable. It also yields a qualitative prediction \((s_{j+1} \geq s_j)\) that is intuitively compelling. However, it suffers from what we (and others) consider to be three major shortcomings. The first limiting feature is the assumption that \(f(x|\theta)\) is known to the DM before
the search commences. Under this *no learning* assumption, the DM is supposed to numerically determine the cutoff values $s_j$ before taking the first observation and extract no information from the observations. But in reality optimal decision behavior is adaptive, and people adjust their criteria for terminating the search as they gather more information during the search.

A second and related shortcoming is that the parameters $\theta$ of $f(x | \theta)$ are known with precision. We are hard pressed to think of examples, even in the case where the sequentially observed objects vary on a single dimension (e.g., sequential search for a good wage). Nor can robustness and “flat maxima” arguments be invoked to justify the full information assumption. The values of $s_j$ are located at the extreme tail of the density function and are consequently very sensitive to even minor changes in the parameter values $\theta$.

The third, and in our opinion the most limiting, feature of the full information approach, which is also shared by the model of Derman et al., is the assumption of a single attribute. As pointed out by Seale and Rapoport (2000), Zwick et al. (2003), and others, there exist sequential search situations where “jobs” can be evaluated and compared to one another in terms of a single attribute (price is the best example) whose distribution function may be known approximately or can be learned with experience. But in most other search situations it may not be possible to specify the value function because the objects have multiple, possibly conflicting, attributes. In a sequential search for an academic position offers are usually compared to one another in terms of starting salary, fringe benefits, availability of research facilities, geographical location, and education facilities for the children, etc. Applicants for a secretarial position vary from one another in their experience, knowledge of computers, social facility, and so on. The tissue type of kidneys for transplantation consists of six, rather than a single, proteins, and perfect matching occurs when the proteins of the donor and recipient are identical (Su & Zenios, 2002). One may possibly propose to extend the full information approach to the multi-attribute case by
postulating some multi-dimensional distribution function. However (see CS), it is difficult, if not impossible, to identify the overall value function \( f(x \mid \theta) \) of multiple attributes \( x \) and estimate its parameters \( \theta \). As we show below, a major attractive feature of the no information approach to modeling sequential search and sequential assignment is that no assumption is made about the value function.

3. Sequential Observation and Selection Problems

The “Secretary Problem” (SP) is a mathematical problem that first appeared in the late 1950’s. Since then, the SP has been extended and generalized in many different directions and given rise to a “field” of study in applied probability theory (see reviews by Ferguson, 1989; Freeman, 1983; Samuels, 1991). The SP has also stimulated experimental, rather than purely theoretical, research on sequential search by Corbin, Olson, and Abbondanza (1975), Seale and Rapoport (1997, 2000), Stein, Seale, and Rapoport (2003), and Zwick et al. (2003).

After surveying the history of the problem and alleviating confusion with other sequential search problems that make strong distributional assumptions, Ferguson proposed the following definition: “A secretary problem is a sequential observation and selection problem in which the payoff depends on the observations only through their relative ranks and not otherwise on their actual values” (1989, p. 284). In a summary chapter, Samuels concluded, “On balance, this seems like the best definition” (1991, p. 381). Although there are many variations and extensions of the problem, in its simple form the Classical Secretary Problem (CSP) satisfies the following assumptions:

1. There is only a single position to be filled.
2. The number of applicants for this position, \( n \), is finite and known to the DM before the search commences.
3. The $n$ applicants are interviewed (observed, inspected) by the DM, one at a time in a random order. Consequently, each of the $n!$ orderings is equally likely.

4. The DM can rank order all the $n$ applicants in terms of their quality (desirability, attractiveness, suitability) from best to worst with no ties. Her decision to either accept or reject an applicant on any given period only depends on the relative rank of the applicant with respect to all the applicants that have already been interviewed.

5. Once rejected, an applicant cannot later be recalled.

6. The DM’s payoff is 1, if she chooses the best of the $n$ applicants (in terms of their absolute ranks), and 0, otherwise. (This objective function is known as “nothing but the best.”)

Managerial and marketing sequential decision and selection problems modeled by the CSP include, among others, selling an asset in the open market, hiring an employee for a job, purchasing some product on the market, assigning a job to a single machine, choosing an investment alternative, or searching to rent an apartment (CS, in press; Zwick et al., 2003). These various search situations are characterized by multi-attribute decision alternatives (“applicants”) that are presented and inspected sequentially over time. Following an evaluation of the alternative, the DM may either choose it and thereby terminate the search or reject it and continue the search for at least one more period. The decision is based on the overall relative rank of the alternative under consideration with respect to its various attributes and the possibility of finding a better alternative if the search is continued.

In the CSP, the DM must assign a single applicant to a single position; all other applicants are not assigned to a position. For most applications, the CSP is too restrictive. Consequently, every one of the assumptions above has been relaxed. Since the generalizations of the CSP are of no importance to the present research, we do not discuss them here. For reviews of these extensions and their optimal decision policies see the references above. The sequential
assignment problem that we describe next encompasses the CSP under a specific formulation—and is, therefore, of greater applicability.

In designing the experiment, we opted to couch the experimental task in terms of “patients” and “hospitals” rather than in the more abstract terms of “applicants” and “positions.” The sequential triage model that we describe below is clearly stylized and fails to capture some essential aspects of the current guidelines for triage assignments. We use the term triage in the common sense of classifying patients according to medical needs and matching of these patients to available care resources. And our sequential model also recognizes that for optimal allocation of resources in the treatment of man-made or natural disasters it is useful to decide as early as possible which patient will benefit most from transport to a dedicated trauma center (Bond, 1997). However, the common guidelines for triaging patients dictate a four-color classification system, where red stands for serious but salvageable life threatening injury/illness, yellow stands for moderate to serious injury/illness, green stands for “walking-wounded” patients who can stand a delay in transportation, and black indicates victims who are found to be clearly deceased at the scene or having fatal injuries. These four “values” clearly differ from the information structure that we assume. But then our purpose has not been to mimic current guidelines directed at well-trained professionals but, rather, to simplify an already complicated sequential decision task presented to naïve subjects.

In the formulation of the sequential selection and assignment problem (SSAP) as a sequential triage assignment, a set of \( h \) hospitals, their capacities, and their costs are assumed to be known by the DM. A set of \( n \) patients arrive to an emergency station in a random order, one at a time, and their “values” (degree of injury), denoted by the random variable \( X_j \), are treated as independent. A major assumption of our formulation sharply differentiates it from the parametric selection and assignment problem. Our formulation is based on the assumption that the DM
officer, who has to assign patients to hospitals, can only rank order patient \( j \) in terms of the severity of her injuries *relative* to the patients who have already been assigned to the hospitals; she cannot determine the severity of the patient’s injuries relative to those patients who are yet to arrive.

Each time the DM inspects a patient \( j \), she must decide to which of the \( h \) hospitals not yet filled to capacity the patient should be assigned. (The script \( j \) corresponds to the number of not-yet-assigned patients.) Each hospital position has an associated “cost” denoted by \( b_k \). The cost \( b_k \) may represent the quality of care and consequently the chances of full recovery, the time of evacuation which increases in the geographical distance of the hospital from the scene of disaster, or some combination of the above. Each patient \( j \) has a “value” score \( x_j \)--a realization of the random variable \( X_j \)--that reflects the severity of her injury. The cost of assigning patient \( j \) to hospital position \( k \) is given by \( f(x_j)b_k \). (In the classical selection and assignment problem, \( f(\bullet) \) is an identity function; \( f(\bullet) \) for the version we study will be discussed below.) The DM’s objective is to assign each patient \( j \) (1, ..., \( n \)) to one of the \( h \) hospitals in order to minimize the *total expected cost* \( \sum_j f(x_j)b_k \). This objective would easily be obtained if the patients were to be assigned to hospitals simultaneously: one merely assigns the smallest \( f(x_j) \) to the largest \( b_k \), the second smallest \( f(x_j) \) to the second largest \( b_k \), and so on. When the patients must be assigned sequentially, and the assignment is based only on their overall value relative to those patients who have already been assigned, the problem becomes non-trivial.

**Terminology and Notation.** The SSAP assumes that there are \( n \) objects (e.g., jobs, patients) that have to be assigned to \( m \) positions (e.g., machines, hospitals). In the triage context that we study, a “position” is interpreted as a bed in a hospital, and an “object” as a patient. Thus, \( m \geq h \) with equality holding only in the case that each hospital has only a single bed. The number of patients
\( n \) could be larger or smaller than the number of positions \( m \). If \( n > m \), then some patients will necessarily be rejected and receive no treatment (or, alternatively, will receive treatment on the spot and released). We assume that there is no cost associated with treating these patients. On the other hand, if \( n < m \), then some of the beds will remain unoccupied. Hereafter, unless otherwise stated, we shall assume with no loss of generality (see CS, in press) that \( n = m \). Each hospital \( s \), \( s = 1, \ldots, h \), has its own capacity \( m_s \); thus, \( m = \sum_s m_s \). In addition, each hospital position \( k \) has an associated cost \( b_k \). Suppose that \( h = 3 \), \( m_A = 3 \), \( m_B = 2 \), \( m_C = 1 \), and that hospitals A, B, and C have associated costs of 150, 50, and 25, respectively. That is, within a hospital, all positions have the same cost. An example with three hospitals with their corresponding capacities and costs is shown in Table 1.

\[\text{--Insert Table 1 about here--}\]

We adopt the convention of ordering the hospital positions in ascending order from the first position of the highest cost hospital to the last position of the lowest cost hospital. This will be useful below when we describe the problem solution formally.

Suppose that among the \( n \) patients, the DM has already observed and assigned to hospitals \( n-j \) patients, and therefore \( j \) more patients are to be assigned. Upon observing patient \( j \), the SSAP assumes that the DM can only determine the relative rank of this patient among the \( n-j+1 \) patients. Thus, the \textit{relative rank} of patient \( j \), which we denote by \( R_j \), is simply her rank among the \( n-j+1 \) patients that have already been observed by the DM. In contrast, the \textit{absolute rank} of patient \( j \), which we denote by \( A_j \), is her rank among all the \( n \) patients (including the ones who have not yet been observed by the DM). In accordance with Assumption 2 of the CSP, all \( n! \) possible sequences of patient arrivals are assumed to be equally likely. Both the absolute and relative ranks are random variables whose realization depends on which of the \( n! \) sequences are
realized. We shall denote their realizations by \( r_j \) and \( a_j \), respectively. Under the assumption that all \( n! \) sequences of patient arrivals are possible, the random variable \( R_j \) follows the discrete uniform distribution

\[
\Pr[R_j = r_j] = \frac{1}{(n-j+1)}, \quad \text{for } r_j = 1, 2, \ldots, n-j+1.
\]

Chun (1996) has shown that the conditional probability of \( A_j \), given that \( R_j = r_j \), is given by

\[
\Pr[A_j = a_j | R_j = r_j] = \frac{\binom{a_j-1}{r_j-1} \binom{n-a_j}{n-j+1-r_j}}{\binom{n}{n-j+1}}, \quad \text{for } a_j = r_j, r_j+1, \ldots, r_j+j-1.
\]

Thus, given the observed value of \( r_j \), the DM can compute the distribution of \( A_j \) as well as its expected value.

**Example 1.** To illustrate the difference between relative and absolute ranks, and the way relative ranks are derived from absolute ranks, consider the following example.

<table>
<thead>
<tr>
<th>Patient Position</th>
<th>1(^{st})</th>
<th>2(^{nd})</th>
<th>3(^{rd})</th>
<th>4(^{th})</th>
<th>5(^{th})</th>
<th>6(^{th})</th>
<th>7(^{th})</th>
<th>8(^{th})</th>
<th>9(^{th})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank ( a_j )</td>
<td>6</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>8</td>
<td>1</td>
<td>9</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>Rank ( r_j )</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>1</td>
<td>7</td>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>

There are 9 patients in this example with absolute ranks \( (a_j) \) from 1 to 9. The absolute ranks \( (r_j) \) reflect the severity of injury, with rank 1 assigned to the most severely injured patient and rank 9 to the least severely injured patient. The patient with absolute rank 6 is the first to be brought to the triage station, the patient with absolute rank 2 is the second, and so on. The DM can only assess the relative ranks (bottom row). Thus, the first patient is clearly the most severely injured of the ones she has seen so far, and is assigned a relative rank 1. The second patient whose injuries are more severe than the first is consequently assigned the relative rank 1. The third patient (with absolute rank 5) is more severely injured than the first (with absolute rank 6) and
less severely injured than the second (with absolute rank 2). Therefore, she is assigned the relative rank 2. This process of assessing relative ranks continues until the ninth and last patient is assigned the relative rank 4, which necessarily corresponds to her absolute rank.

After the DM observes the relative rank \( r_j \) of the \( j \)th patient, she must decide to what hospital to assign this patient. Associated with the \( k \)th position (bed in the hospital) is the cost \( b_k \), \( k=1, 2, \ldots, n \). With no loss of generality, we assume that the positions are arranged such that \( b_1 \geq b_2 \geq \ldots \geq b_n \). For example, supposing that there are three major hospitals A, B, and C, where the maximum capacities are 2, 3, and 4, positions, respectively. Then, the costs of these hospitals would be \( b_1 = b_2 \geq b_3 = b_4 \geq b_5 = b_6 = b_7 = b_8 = b_9 \). If a patient with absolute rank \( a_j \) is assigned to the \( k \)th position, then the SSAP assumes that the cost \( a_j b_k \) is incurred. (In the example in Table 1, \( b_1 = b_2 = b_3 = 150 \), \( b_4 = b_5 = 75 \), and \( b_6 = 25 \).)

4. The Optimal Assignment Policy

The objective of the DM is to ensure that the available medical resources are used to treat the greatest number of patients in need of these resources. In our formulation, this is tantamount to minimizing the total expected cost. The commonly used procedure of “first come, first assigned to the nearest hospital” is, in general, not an effective assignment procedure. It may result in greater mortality (i.e., “cost”) because some of the patients who do not require immediate medical treatment receive it because they happen to arrive earlier. Consequently, more seriously wounded patients who arrive later and legitimately require the treatment do not receive it. The procedure “first come, first assigned to the better (and most costly) hospital” will, in general, not be an effective assignment system either. Effectiveness under this procedure will only be achieved if the most severely wounded patient arrives first, the second most severely wounded patient arrives second, and so on.
Denoting a particular assignment of the $n$ patients to the $m$ positions by $\Gamma$, where $\Gamma$ is a permutation of the integers from 1 to $n$, the DM would like to find $\Gamma^* = \arg \min_{\Gamma} \left( \sum_{j=1}^{n} a_j b_k \right)$. However, since the patients arrive sequentially, $\Gamma^*$ cannot easily be found. Instead, given the constraints of the problem, the optimal decision policy minimizes the expected value of the total cost: $E \left( \sum_{j=1}^{n} a_j b_k \right)$. A decision policy dictates to which open position to assign the $j^{th}$ patient when her relative rank is $r_j$.

Using dynamic programming, CS constructed an optimal assignment scheme that takes into account the severity of injury and the location, capacity, and medical facilities available at each of $h$ hospitals. To describe it, define the critical relative rank $r_{i,j}^*$ at stage $j$ by

$$r_{i,j}^* = \begin{cases} 0 & \text{if } i = 0 \\ \frac{1}{n-j+3} \sum_{k=1}^{n-j+2} \min \{ \max(r_{j-1}, r_{i-1,j-1}), r_{i,j-1}^* \} & \text{if } 1 \leq i \leq j \\ \infty & \text{if } i = j \end{cases}$$

Equation (1) shows that the critical relative ranks do not depend on the costs $b_k$ except through their order. They satisfy the condition

$$0 < r_{0,j}^* < r_{1,j}^* < \ldots < r_{j,j}^* < \infty.$$ 

Based on the critical relative ranks, the optimal assignment policy at each stage $j$ is to assign the patient of relative rank $r_j$ to the $i^{th}$ position if $r_{i-1,j}^* < r_j < r_{i,j}^*$.

**Example 2.** Consider the same parameter values as in Example 1 with $n=9$ patients who arrive in the order

$$\{a_j\} = (6, 2, 5, 3, 8, 1, 9, 7, 4).$$

As shown in Example 1, the corresponding relative ranks observed one at a time are

$$\{r_j\} = (1, 1, 2, 2, 5, 1, 7, 6, 4).$$

The critical relative ranks for Example 2, computed from Eq. (1), are presented in Table 2.

Continuing with this example, assume that there are three hospitals, A, B, and C, with a corresponding number of positions (beds) 2, 3, and 4. Recall that \( j \) indicates the number of patients remaining to be assigned. Consider the first patient (\( j=9 \)). This patient has absolute rank 6 and the corresponding relative rank 1 is between \( r^*_{4,9}=0.9153 \) and \( r^*_{5,9}=1.0847 \) at stage 9. Therefore, the first patient is assigned to position 5 (in hospital B). Following this assignment, hospital B is left with only two beds. Consider next the second patient (\( j=8 \)) of absolute rank 2 and the corresponding relative rank 1, which is between \( r^*_{2,8}=0.9585 \) and \( r^*_{3,8}=1.2460 \) at stage \( j=8 \). This patient is assigned to position 3 (also in hospital B). This leaves hospital B with only a single bed. Continuing in this way, the optimal assignment of the patients to the nine positions is

\[ P = (5, 3, 6, 4, 9, 1, 8, 7, 2), \]

or in terms of the three hospitals:

\[ A = \{1, 4\} \quad B = \{2, 3, 6\} \quad C = \{5, 7, 9, 4\}. \]

This optimal assignment is depicted in Table 3 that shows the absolute ranks of the nine patients. Note that this is not the best possible assignment. If all nine patients were to be assigned \textit{simultaneously} rather than sequentially, the optimal assignment procedure would be to assign patients with absolute ranks 1 and 2 to hospital A, patients 3, 4, and 5 to hospital B, and the remaining four patients with absolute ranks 6, 7, 8, and 9 to hospital C.

\[ A = \{1, 4\} \quad B = \{2, 3, 6\} \quad C = \{5, 7, 9, 4\}. \]

--Insert Table 3 about here--

\textit{Alternative Assignment Procedures.} Supposing that the costs per position in hospitals A, B, and C are 250, 100, and 40 units, respectively. Then, the total cost associated with the optimal assignment policy in Table 3 is

\[ 250 \times 5 + 100 \times 11 + 40 \times 25 = 3510. \]
If, for example, patients are assigned to the three hospitals in the order of their arrival, the resulting assignment is
\[ A = \{6, 2\}, \quad B = \{5, 3, 8\}, \quad C = \{1, 7, 9, 4\}, \]
and the total cost associated with this assignment procedure is
\[ 250 \times 8 + 100 \times 16 + 40 \times 21 = 4440. \]
The total cost of this assignment procedure is 26.5\% higher than the one associated with the optimal assignment procedure. The worst possible assignment procedure is when the least injured patients are assigned to hospital A and the most seriously injured to hospital C:
\[ A = \{8, 9\}, \quad B = \{5, 6, 7\}, \quad C = \{1, 2, 3, 4\}. \]
The total cost associated with this worst possible assignment procedure is
\[ 250 \times 17 + 100 \times 18 + 40 \times 10 = 6450. \]

**CSP as a Special case of SSAP.** Chun and Sumichrast (in press) have shown that the CSP is a special case of the SSAP with the following cost function:
\[ b_k = \begin{cases} 1 & \text{if } k = 1 \\ 0 & \text{if } k > 1 \end{cases} . \]
In other words, this is the case where there is only a single position \((m=1)\) available for many sequentially arriving patients, and the DM’s task is to minimize the absolute rank of the patient chosen for this privileged position.

### 5. Experiments 1 and 2: Initial Experimental Studies of the SSAP

**Overview.** Chun and Sumichrast have developed a recursive procedure for determining optimal assignment in the SSAP; here, however, we are interested in actual assignments of financially motivated DMs. We had subjects perform the SSAP in a computerized decision task that simulates sequential triage assignment following a fictitious disaster. Repeating the task for multiple trials, our design allows us to assess the subjects’ assignments relative to the optimal
decision policy and the changes in behavior with experience. We first conducted an experiment in which the number of patients was relatively small (n=12); then, to test the generality of our findings, we conducted a second experiment in which this number was doubled (n=24).

Method

Subjects. Thirty-six financially motivated subjects participated in Experiment 1 and thirty-nine in Experiment 2. The experiments were conducted in the computerized Economic Science Laboratory at the University of Arizona. All the subjects were University of Arizona students recruited by advertisements asking for volunteers to participate in a decision making experiment with payoffs contingent on performance. The mean payoff per session, that typically lasted 1 to 1.5 hours, was about $25 (minimum $15, maximum $40). In addition to the monetary payoff, the subjects received class credit for their participation, if they requested it.

Procedure. Each subject was provided with a written (hard-copy) of the instructions for the SSAP task. The general task was described as follows:

The experiment is concerned with the decision making process following a natural or man-made disaster (e.g., earthquake, fire, terrorist attack). When the number of people injured overwhelms the medical resources of area hospitals, there is typically a central emergency allocation system that handles the flow of patients. We have designed an experiment that simulates emergency situations whose purpose is to study how people make sequential assignment decisions.

The subsequent instructions placed special emphasis on the computation of relative rank information for the patients and on the way in which assignment costs were to be determined. Several examples of the SSAP were presented in detail. The first involved a simple assignment problem in which three patients had to be assigned to three hospitals, each of which had one position. The costs of each of the six possible assignments were presented to the subject, along
with a detailed explanation of how they were computed. Two more examples were then presented to the subjects, each involving six patients. Finally, the instructions explained the payoff scheme. Specifically, the subjects were told that they would randomly select three trials for payment at the conclusion of the task by drawing numbers from a hat. Their payments for each trial would be based on their net earnings for that trial (Net Point Earnings = Endowment Points - Cost Incurred on Trial). The total points earned on these three trials would then be converted to a cash payment at a rate of $1 per 1000 points.

After reading the instructions, two practice problems were presented to verify the subject’s understanding of the task. The experimental problems were presented once the subjects successfully completed these two practice problems. Each subject completed 60 actual trials (replications) of the SSAP.

**Manipulations.** We conducted two separate experiments that varied the number of patients: \( n=12 \) (Experiment 1) and \( n=24 \) (Experiment 2). The number of hospitals was fixed at \( h=4 \). The number of openings for the hospitals in Experiment 1 (\( n=12 \)) was as follows: \( m_1=3, m_2=4, m_3=3, \) and \( m_4=2 \); for Experiment 2 (\( n=24 \)), it was \( m_1=6, m_2=7, m_3=6, \) and \( m_4=5 \). The costs for all positions within a particular hospital were the same in both experiments. Hospital 1, the most costly one, had an associated cost of 100, and Hospitals 2, 3, and 4 had associated costs of 50, 25, and 10, respectively. Participants in the \( n=12 \) case received an endowment of 10,000 units ($10) on each trial; those in the \( n=24 \) case received 20,000 units ($20).

Each trial consisted of a SSAP with \( n \) (12 or 24) patients, each presented separately on a different *period*. Each trial was structured in the same way: The subject was sequentially shown the relative rank for each patient and required to assign the patient to an open hospital position. Subjects assigned a patient to a hospital by clicking on the button for that hospital. The patient’s
ID--simply her number in the sequence of $n$ patients--was then placed in the open position for that hospital and remained there until the termination of the trial. Each hospital button gave the hospital name and the cost associated with that hospital (e.g., “Hospital 1 (cost = 100”)). Hospital buttons were “dimmed” and could not be clicked whenever the hospital was full. Once placed in a particular hospital, a patient could not be moved to another hospital.

To allow comparison between subjects, in both experiments we generated a sample of 60 different random sequences of patients (out of a population of $60!$ sequences). In each experiment, each subject was presented with the same 60 sequences in the sample, but the presentation order was randomized across subjects.

**Results**

*Overview.* Using Equation 1, for each trial and each subject separately we computed the hospitals to which each of the $n$ patients should be assigned. Recall that the optimal assignment algorithm instructs the DM into which *open position* a patient should be put. In our task, however, subjects could only assign patients to *open hospitals* (i.e., open hospitals with open positions); they *could not* select the bed in a particular hospital a patient should be placed. Therefore, we simply mapped the optimal assignment positions (integers from 1 to $n$) given by Equation 1 to hospitals (1 to 4). For example, if the third open position is in Hospital 2, then an assignment to (any position in) Hospital 2 is optimal; assignments to any other hospital are not.

We define *unconditionally optimal assignments* of the $n$ patients as the $n$ assignments dictated by Equation 1. Given the arrival ordering of the patients, these can be computed *a priori* without consideration to the subject’s assignments. On the other hand, *conditionally optimal assignments* are based on the subject’s previous assignments. Specifically, the assignment of patient $j$ is *conditionally optimal* if and only if she is assigned to the optimal open hospital (as determined by Equation 1), given the availability of open hospital positions, which are
determined by the vacancies left after the patients previously assigned by the subject. Clearly, unconditional and conditional assignments coincide if and only if the subject adheres to the optimal policy on each period. All the results reported below are based on comparisons to the conditionally optimal assignments.

**Quality of Patient Assignments.** The subject’s performance in the SSAP was measured by the root-mean-squared-deviation (RMSQ) of the actual assignments of the $n$ patients from the (conditionally) optimal assignments. Denoting the hospital to which the $j^{th}$ patient would be optimally assigned $s^*_j$, and the hospital to which the patient was actually assigned $s_j$, the subject’s performance can be assessed by the measure

$$\text{RMSQ(Actual, Optimal)} = \left[ n^{-1} \sum_{j=1}^{n} (s_j - s^*_j)^2 \right]^{.5},$$

which is 0 for a perfectly optimal assignment solution (conditional or unconditional) and grows as the quality of the assignment decreases.

Figure 1 shows the average RMSQ scores across the 60 experimental trials for Experiments 1, 2, and 3. (The results for Experiment 3 will be presented later.) Three characteristics of the performance scores are noteworthy. First, the RMSQ score is non-zero across all 60 trials, indicating suboptimal solutions to the SSAP. Secondly, however, we observe that the RMSQ scores in both experiments slowly decrease across trials, indicating learning. To establish an alternative baseline for performance (i.e., one other than optimal), we computed the expected RMSQ under fully random assignment to open positions. For the configurations used in Experiments 1 and 2 the expectations are 1.27 and 1.42, respectively, both of which are significantly greater than the observed RMSQ in each experiment. Despite the complexity of the task—the computations required for optimal assignment are non-trivial—the subjects in both
experiments seem to have developed decision heuristics that allowed them to make reasonably
good assignments and also learned to improve their assignments with experience. Finally, there
is considerable subject heterogeneity in the RMSQ measure. Frequency distributions of
individual mean RMSQ scores (i.e., RMSQ scores averaged over all 60 assignment problems for
each subject) in Experiments 1, 2, and 3 are exhibited in Figure 2.

Figure 3 displays the mean difference between optimal and actual assignment positions
across the $n$ periods (patient positions) for each subject in Experiments 1 (top panel), 2 (middle
panel), and 3 (bottom panel). Although there is some variability in the plots, the general pattern
across the $n$ positions is quite similar for most subjects. Specifically, $s_j - s^*_j$ is quite often
positive (and is positive on average, see Fig. 3) for the first patient. This indicates a slight bias to
place the first patient in a worse hospital than she ought to be placed in. The differences then
tend to be negative for the next few patients before converging to 0 for roughly the last half of
patients. For the configuration used in Experiment 1, the expected value of $s_j - s^*_j$ under random
assignment to open positions is positive for the first patient and negative for patients 2 through
11. The expectation is zero for the final patient under all possible assignment strategies, using the
conditionally optimal criterion. The expectation under random assignment is also positive for the
first patient in Experiment 2, but is then (very close to) zero for patients 2 through 23. Hence, the
positive $s_j - s^*_j$ difference for patient 1 in these experiments may be an artifact of the particular
configurations used. Further, we should be cautious in drawing strong inferences from the
negative $s_j - s^*_j$ for early patients in Experiment 1: this result could obtain for a variety of
reasons. However, we can say more about the $s_j - s^*_j$ results in Experiment 2; specifically, there
is a pronounced bias to over-assign early patients to good hospitals, i.e., to overtriage early

--Insert Figs. 1 and 2 about here--
patients. The generality of this bias will be addressed in Experiment 3, where we used multiple hospital configurations.

--Insert Fig. 3 about here--

Assignment Probabilities. For each experiment across the $n$ periods we computed the proportion of times (or probabilities) that a patient was assigned to the $s^{th}$ hospital by the subjects (actual) and by the (conditionally) optimal policy. Results for Experiments 1 and 2 are depicted in Figs. 4 and 5, respectively. These plots are exceptionally revealing and consistent across experiments. First, note that the optimal assignment probability to Hospital 1 for period 1 is 0 for both conditions. However, the subjects assigned the first patient to Hospital 1 with probability about 0.20 for the $n=12$ case (Experiment 1) and about 0.15 for the $n=24$ case (Experiment 2). In both experiments, the first patient should always be assigned to Hospital 2. Subsequent actual assignment probabilities for Hospital 1, however, are quite close to the optimal assignment probabilities, though in both experiments they are generally slightly smaller. With the exception of the first patient, in both experiments patients are generally assigned to Hospital 4 less often than they ought to be. Instead, there is a tendency to place patients in Hospitals 2 and 3 with higher probability than is dictated under the optimal policy. We tested the difference in actual and optimal assignment probabilities for positions 1 to $n-1$ for each hospital in Experiments 1 and 2 with sign tests, using sign (optimal probability – actual probability). In Experiment 1, the actual assignment probability to Hospital D was less than the optimal for 9 of the 11 positions, $p<0.05$; for Hospital A, the difference was positive for 6 of 11 positions, $p>0.05$. The probabilities of negative differences for both Hospitals B and C were statistically significant, $p<0.05$. All tests for the data in Experiment 2 were significant and in the direction predicted by the middleness bias: that the subjects tend to avoid the best and particularly the worst hospitals
more often than they ought to, preferring instead to assign patients to intermediate (or middle) hospitals.

--Insert Figs. 4 and 5 about here--

Discussion

Assessing the absolute quality of the subject’s performance is not possible, as no obvious standard exists other than the optimal and random assignment. We observe that our subjects’ assignments are considerably better than random assignment; however, this is a weak criterion. Perhaps better bases for determining mean performance are the assignment probabilities displayed in Figs. 4 and 5, which show that the difference between actual and optimal assignment probabilities tends to be relatively small. Although it is difficult to assess the absolute quality of assignments, we can clearly see that the assignments show very systematic departures from optimality (using the assignment probability metric). In particular, we see what we refer to as a middleness bias, namely, a propensity to place patients in the intermediate quality hospitals. This bias is consistent with subjects saving the extreme hospitals (the best and worst) for patients in whose severity of injuries they have more confidence, viz. those who arrive later in the sequence. Further, we see that the DMs learn and make better assignments with experience.

6. Experiment 3: Testing the Generality of the Results of Experiments 1 and 2

Overview. A potential shortcoming of Experiments 1 and 2 is that our conclusions may be confounded by the particular hospital configurations that we used. In both experiments, the hospital configuration was fixed for all 60 trials. Further, the configuration in Experiment 2 was similar to the one in Experiment 1; the only difference was that each hospital had three more positions in Experiment 2. To overcome this shortcoming, in Experiment 3 we varied the configurations of hospitals from trial to trial. Doing so allows us to determine whether the
behavioral patterns observed in the previous experiments are general biases that subjects exhibit in the SSAP. In addition, we can test whether the learning that was reported in the previous two experiments is due to subjects learning to assign patients to a particular hospital configuration. Thus, Experiment 3 provides a test of the generality of our previous conclusions that are not limited to a fixed hospital configuration.

Method

Subjects. Thirty-seven financially motivated subjects participated in Experiment 3. As before, the experiment was conducted at the Economic Science Laboratory at the University of Arizona and the subjects were recruited in the same fashion. The mean payoff per session was about $31 (minimum $15, maximum $40). In addition to the monetary payoff, the subjects received class credit for their participation if they requested it.

Procedure. Except of changing the hospital configurations across trials, the procedure was identical to the one used in the previous two experiments.

Manipulation. The subjects were presented with five different hospital configurations each repeated twelve times. Each subject observed the same random orderings of subjects for the twelve repetitions of each configuration, but the ordering of trials was randomized separately for each subject. We set \( n = 24 \) for all configurations. Each subject received an endowment of 20,000 units on each trial. Again, the conversion rate was $1 for every 1000 point earned on three randomly selected trials.

Table 4 presents the five hospital configurations used in Experiment 3. These cover much of the space of possible configurations: positions in Configuration 1 are uniform over the hospitals; positions in Configuration 2 are peaked in the center, whereas those in Configuration 3 are peaked on the extremes; Configuration 4 (5) has a negatively (positively) skewed distribution of openings.
Results

Overview. As for Experiments 1 and 2, the results are based on the conditionally optimal assignments.

Quality of Patient Assignments. Figure 1 shows that the quality of patient assignments—as measured by the RMSQ scores—is of the same order of magnitude as observed in Experiments 1 and 2. (The expected value of RMSQ under random assignment for the configurations used in Experiment 3 is 1.46.) In all three experiments, the RMSQ scores decrease across trials at roughly the same rate. The distribution of RMSQ over subjects in Experiment 3 is shown in the bottom panel of Fig. 2.

The mean difference between actual and optimal assignments (averaged over configurations) tends to be negative for early arriving patients and then converges to 0 (see Fig. 3). The expected value of $s_j - s_j^*$ under random assignment is -0.25 for patient 1 and near 0 for patients 2-23. We can, therefore, be confident that the consistently negative $s_j - s_j^*$ for early patients represents a genuine assignment bias and is not an artifact of the problem structure. Further, we can now confidently conclude that the positive difference for the first patient in Experiments 1 and 2 is, indeed, an artifact of the configurations used in those experiments. Thus, early patients tend to be overtriaged: on average they are placed into better hospitals that should be reserved for other potentially more severely injured patients.

Assignment Probabilities. Mean actual and optimal assignment probabilities (averaged over the five configurations) are exhibited in Fig. 6. Comparison of Figs. 4, 5, and 6 shows the same middleness bias: on average, patients are assigned to the intermediate hospitals (B and C) with higher probabilities than they ought to be. We used sign tests to test the assignment probabilities
for each hospital in each configuration. From the middleness bias observed in Experiment 1, we predict that the actual assignment probabilities to Hospitals A and D will more often be less than those of the optimal assignment strategy and that the actual probabilities will more often be greater for Hospitals B and C. These predictions were statistically upheld for all hospitals in configurations 1, 3, 4, and 5 and also for two of the four positions in configuration 2. Figure 7 exhibits the aggregate (averaged) results from all five configurations. Configuration 2, which provides the least support for the middleness bias prediction, has the greatest number of positions in the middle hospitals. It seems that the structure of the problem with Configuration 2 accommodates or corrects for the middleness bias. In sum, then, the middleness bias does seem to be a general property of subjects’ assignments in the SSAP, except in cases in which the greatest number of open positions is in the intermediate hospitals, in which case the structure of the problem compensates for the bias.

An additional observation is in order. For all configurations, the subjects assigned the first patient to the best hospital (Hospital A) with non-zero probability, though under optimal assignment for all five configurations the first patient should never be assigned to hospital A. Although we observe a general learning trend across trials, the first patient error tends to persist. The probability of assigning the first patient to Hospital A in the first 30 trials is 0.08 and in the last 30 trials is 0.10. The difference between the two is not significant.

Discussion

The consistency of the results across Experiments 1-3 is striking. The major findings can easily be summarized. First, there is a general bias to assign patients to the two middle hospitals with greater probability than is dictated by the (conditionally) optimal policy. Subjects seem to be trying to reserve the best and worst hospitals for later patients, about whom they will have
more information (as relative ranks are more informative about absolute rank for patients later in the sequence). The second finding, which complements the first, is that there is a bias to place early patients into hospitals that are better than the ones to which they ought to be assigned. Put differently, early patients tend to be overtriaged. Taken together, the first and second findings reveal that there is a bias to over-assign patients to middle quality hospitals, and further that the middleness bias is in the direction of the better middle hospital. Finally, and most importantly, we observe learning across repeated play of the SSAP: The subjects’ assignments move closer to the optimal assignments with experience.

7. Implications and Future Directions

Using naïve subjects, rather than individuals trained in triage assignment, we observed and documented systematic biases in triage assignments. After careful consideration, and following the practice used in most previous experimental studies of dynamic decision making (e.g., firefighting by Brehmer & Allard, 1991; vehicle navigation by Jagacinski & Miller, 1978; supervisory control by Kirlik et al. 1993; health management by Kleinmuntz & Thomas, 1987), we opted to use a scenario to which inexperienced subjects may easily relate rather than an abstract scenario of assigning “applicants” to “positions.” However, the possibility that the systematic bias detected in the present study is in part related to the particular features of the cover story we use can only be ruled out by subsequent studies that vary \( n, m \), and the way they are presented to the subjects.

There are two important features of our decision problem that may not be characteristic of many sequential assignment problems. First, the DMs in our experiments were informed of the number of to-be-assigned patients. A more realistic assumption is that the DM can only estimate the distribution of \( n \) (see Seale & Rapoport, 2000, who relaxed the assumption of known number of applicants in an experimental selection task). Currently, the optimal assignment policy for this
problem is not known. Another possible extension is to cases in which the DM must assign batches or groups of applicants simultaneously to particular positions; for example, this is relevant when patients must be transported to hospitals in ambulances or helicopters. The optimal policy for this class of problems is yet to be derived.
References


Acknowledgement

We gratefully acknowledge financial support by a contract F49620-03-1-0377 from the AFOSR/MURI to the University of Arizona.
### Table 1. Three Hospitals with a Total Capacity of Six Beds

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<th>Hospital B (Cost = 50)</th>
<th>Hospital C (Cost = 25)</th>
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<td>Bed 3 (3)</td>
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Note that the numbers in parentheses represent position number.
Table 2. Critical Relative Ranks for the Sequential Assignment Problem in Example 2

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Table 3. Optimal Assignment to Hospitals in Example 2.

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Table 4. Hospital Configurations Used in Experiment 3.

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